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MECHANISMS OF "SUPERRESOLUTION" IN DYNAMIC MEASUREMENTS

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The effects of simultaneous influence of discretization and quantization on the evaluation of the result of an indirect measuring experiment are analyzed with a simple numerical example.

The writing of this paper has been encouraged by the persistent contradiction between theoretical models describing the procedures of measuring experiment in thermophysical experiments. The influence of the evolution of instrumentation technology has stimulated the abandonment of stationary methods in favor of dynamic ones in thermophysical measuring experiments, the passage to more complex models of a measuring cell, and the use of predominantly digital computers in manufacturing measuring devices. At the same time, the procedure used for evaluation of measurement errors is based on the model of *statistical* processing of *direct* multiple measurements of an *analog stationary* quantity. Quantized-digitized-signal models used in processing measurement information have entirely been borrowed from communication theory, where there were serious historical prerequisities for the formulation, development, and reduction of these models to canonical form precisely in existing form, and the distinctive features of measuring experiment were allowed for in no way at all. This brings about substantial problems in attempting to identify the parameters of the models (of any complexity) of dynamic processes in a measuring cell by digital experimental data. Forcing the realistic models of measuring experiment to fit standardized procedures primarily causes the devices to lose their resolving power.

However, the last decade has seen reports on the creation of measurement procedures and devices possessing a "superresolving" power. What this means in the general case is that the method proposed enables one to obtain a real resolution of the estimate of the quantity measured noticeably higher than that allowed by the methods reduced to canonical form. Technical implementation is basically the same, whereas the effect is attained due to the more careful analysis of the models of components of the measuring process [1–7].

The present work seeks to analyze the basic mechanisms of production of the effect of superresolution as applied to dynamic measurements in general and their variants used in thermophysics in particular.

Models. Since we are dealing, in the paper, with a number of problems having a conceptual character as far as measuring-experiment models are concerned, numerical experiment directly illustrating both the problem and the mechanism of gaining in resolving power will be the basic technique of investigation.

The experiment in question is based on the model interpreting measurement information which is presented in digital form, i.e., that discreted and quantized simultaneously. Discretization and quantization operations are an integral part of modern measuring experiment. The method of using them and the purpose significantly differ from how they are used for communication purposes. However the models of these operations have been taken from communication theory without allowing properly for the distinctive features of problems of measuring experiment. This is also true of modern educational courses (see, e.g., [8–10]).

The main difference between the formulations of measuring-experiment and communication problems is that the former seeks to estimate the running value of a parameter (e.g., of the signal observed) and not to restore the initial signal, as in communication applications. It is convenient to represent the estimate in the form of a confidence interval or an uncertainty interval. The aim may be considered to be obtained if the parameter sought is directly measured and discretization is carried out exactly at the necessary instant of time, with the quantum being fairly small. All these conditions are far from being always observed simultaneously.

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The above discussion is conveniently formalized as follows. At the instant of time t, we sample the measured quantity y. Its value is estimated by a certain grid with a quantum ν (for a regular analog-to-digital converter, we have $\nu = U/2^n$). The result of this experiment is represented in the form $([y], [t])$, where the estimates of the measured quantity and the instant of sampling are given as interval numbers whose uncertainties are the quantum and the period of discretization respectively. If the accuracy of the result and the instant of sampling cannot be considered to be satisfactory or the parameter measured is x rather than y (here, we have the dependence $y = f(x, p)$, which is generally nonunique, nonlinear, and involving other parameters (p)), we may use the sequence of samples $\{([y], [t])\}$. Under favorable conditions (with the corresponding organization of measuring experiment and an adequate method of processing), we are able to improve the estimate of the quantity sought due to the information contained at different sites of this sequence of samples. We are dealing with the evaluation of the quantity sought by experimental data in indirect experiment.

The model of using a sequence of discrete and quantized samples appears simple: $\{([y], [t])\} \to f^{-1}(x, t) \to [x]$, but its realization requires algorithms for efficient conversion of interval data. Such an algorithm has been described in [11], and a comparative analysis of several methods of estimation of the values of the measured parameters by discrete and quantized data in indirect experiment has been given in [12].

A simple but general model of a dynamic process — a measurement-information carrier — may be obtained from the following condition: for the dependence $y(t)$ with $dy/dt \neq 0$; then, taking $dy/dt = x$, we obtain $y = xt + b$. When $x = 0$ the measurements are static; when $x \neq 0$ they are dynamic.

Results. The influence of discretization and quantization on the results of measuring experiment will be shown with a simple idealized example. We consider an indirect experiment performed according to the model $y = x(t - c) + b$. This is the simplest model with one unknown parameter — the absence of noise and nonlinearities — but quite an interesting result. The rate of change in the quantity (signal) y observed directly is measured. This quantity is converted to a sequence of numbers by means of an analog-to-digital converter. The code of the sampling time is made to correspond to each number. Since in most cases the uncertainty of the counting of time is much smaller than the period of sampling, the pair of numbers obtained will be interpreted, for the sake of simplicity, as a sample with an exact time $([y], t)$. The numerical experiment will be carried out for quantities expressed in relative units. The parameter b will be considered to be known and (for the sake of definiteness) close to zero.

We investigate the dependence of the uncertainty of evaluation of the measured parameter $\Delta[x] = x_{\max} - x_{\min}$ on its actual value. The measurement procedure is modeled as follows. A certain portion of variation in the signal y is observed, on which we take all the samples. The dynamic range of the analog-to-digital converter is selected so as to prevent the signal from leaving it even for the maximum values of the parameter measured. For the sake of simplicity, both x and the dynamic range of the analog-to-digital converter are selected to be equal to 0 ... 1; consequently, all the measurements are carried out in a time interval of 0 ... 1 divided into m samples. The uncertainly of the experimental result $\Delta[x]$ is interpreted as the range of all the x values for which the analog-to-digital converter generates the same code. Since all parameters of the model are normalized, the uncertainty of evaluation of the quantity sought turns out to be normalized, too. The result of the numerical experiment is shown in Fig. 1.

An analysis of the result obtained reveals several features. We may obtain the worst estimate of the uncertainty, assuming that it is determined by one sample located at the most optimum site. In this example, this is the last sample for the maximum value $\Delta[x]_n = \nu = 0.016$, which corresponds to the height of the left step in Fig. 1. The fact that all the results for the sequence of samples lie lower than the worst estimate is not a surprise; the fact that such a large number of results approaches it is more strange.

We may compute the best estimate, assuming that all the samples make an analogous contribution to the result, thus improving it in proportion to the number of samples, which is quite true. This is the limiting case of improvement of the estimate of a measurement result and consequently the best case of resolving power for regular methods of processing of measurement information. For the example in question, the best estimate is $\Delta[x]_b = \nu m^{-1} = 1.563 \cdot 10^{-4}$ (it is marked by the line with a "b" in Figs. 1 and 2). Surprisingly, we observe results with an uncertainty smaller than that with the best estimate, and the fraction of these results is not very small (Fig. 2).

The standard (statistical) estimate of the uncertainty is somewhat between those two given above; its specific location somewhat depends on how the sample quantum is interpreted and what kind of statistics is assigned to it. Since this is an example of forcing the reality very roughly to fit a standard model, this error source is commonly

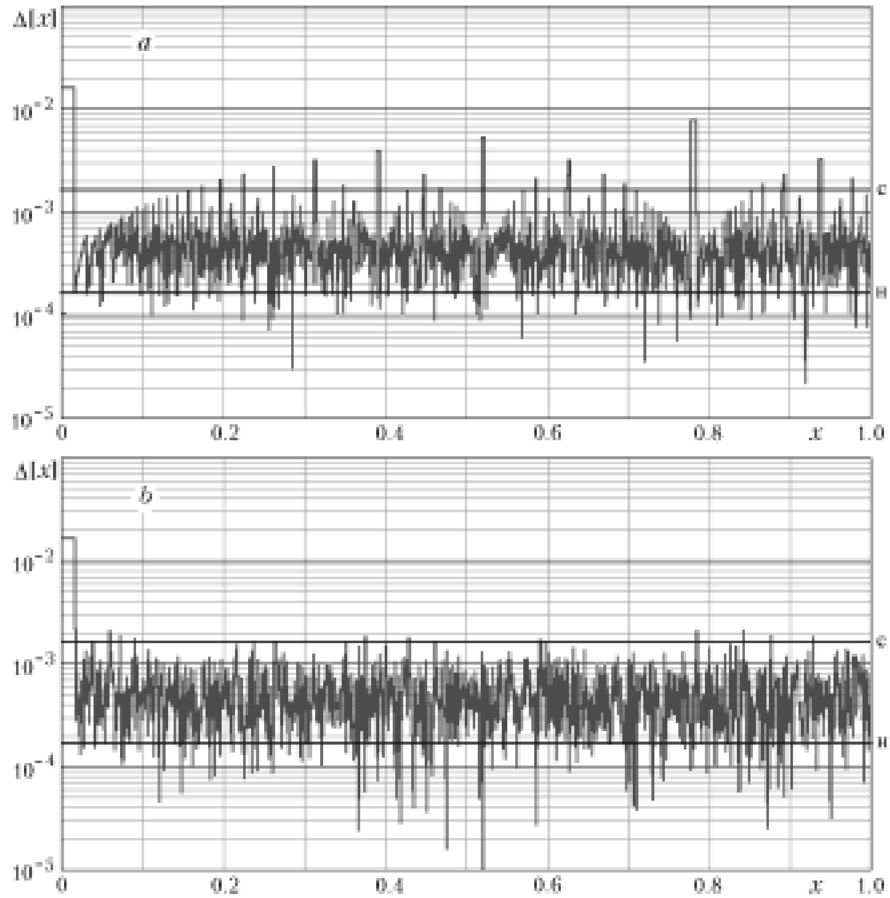

Fig. 1. Diagrams of uncertainties for evaluation of the coefficient of linear dependence for the capacity of the analog-to-digital converter $n = 6$, the number of samples $m = 100$ taken uniformly (a) and randomly (b), the constant component of the process $b = 0$, and the initial delay of the sequence of samples $c = 0.0051$.

considered to be quantization noise having an amplitude $\nu/2$ and the statistics of a uniform distribution. When the number of samples is large, it is assumed that the confidence interval will differ only slightly from the result of the calculation performed for the normal distribution of random quantities. Since the Student coefficient is close to unity for large m, we obtain $\Delta[x]_s \approx \nu m^{-1/2} = 1.563 \cdot 10^{-3}$ (denoted by s). It is remarkable that this is a good upper estimate (bound) of the uncertainty but only for randomized samples (Fig. 1b). What this means is that if the uncertainty of quantization is treated as random noise, a considerable amount of information obtained in measuring will be lost.

The quantum uncertainty is not noise for another reason — that one can obtain an error substantially higher than the statistical threshold even for an ideal process. Here the mechanism is the same as that in obtaining "superresolution" — a fine interaction between the bounds of the sample's value, the sampling time, and the trajectory of the process.

The effects in question are similar to aliasing and moiré; consequently, we can fight them in the same manner — by randomization. The simplest method of randomization is to sample at randomly selected instants of time and not uniformly. The result of a randomized numerical experiment is given in Fig. 1b, where we easily notice a decrease in the number of very poor results. The fraction of very good results somewhat increases, but their distribution remains previous, on the whole.

An analysis of even such a simple problem brings about problems of methodological and applied character. The former refer to the subject and procedure of investigation of the above features for both particular and more gen-

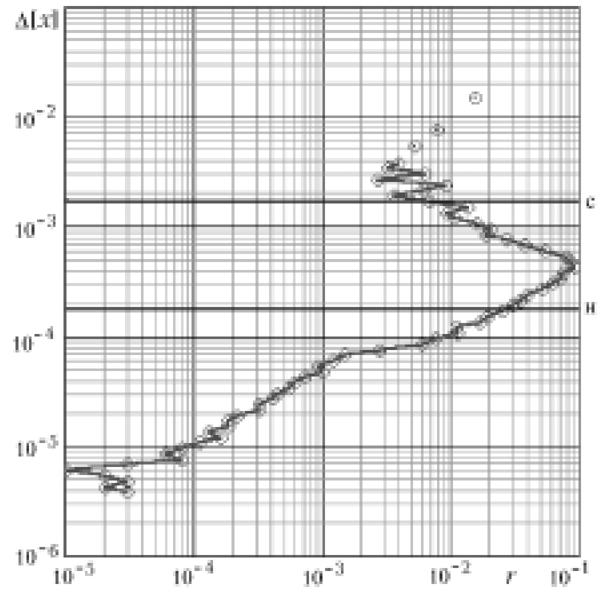

Fig. 2. Histogram of uncertainty distribution by value, obtained from the data of Fig. 1a.

eral cases. From the viewpoint of applications, it is necessary to elucidate the extent to which the features mentioned are useful or dangerous.

Noteworthy is the fact that neither the model of an object nor the model of a measurement procedure contain any assumptions of random processes. The idealized models in question possess an absolute recurrence period. This is also true in the case of a random arrangement of samples, if this arrangement is remembered with time and is exactly repeated again. The complexity of the result is similar to a randomness, a sequence of figures of an irrational number.

It seems impossible to investigate the resulting fractal-like dependences by analytical means. Numerical experiment remains the only available tool of investigation. It is expedient to use it in designing a new experiment, if an unsatisfactory uncertainty of the result is obtained, and the observed level of noise enables one to hope to improve the result. In this situation, even a simple change in the rate of sampling may substantially change the result. Just a small decrease in the rate may turn out to be sufficient. On the uncertainty diagram, such a change will cause a certain displacement of peaks.

In practice, one can obtain "superresolution," i.e., resolution above the best-estimate threshold, only on condition that the signal/noise ratio is very high. It is precisely the best results that primarily become worse upon the appearance of even a relatively low noise. The exaggerated situation is that the noise threshold "forces" the results out to the domain of a higher uncertainty with gradual decrease in the signal/noise ratio, beginning with the best results and subsequently gradually involving the remaining results immediately before the threshold. Conversely, no special efforts are required to overcome the statistical threshold, since it represents the upper bound. Obtaining "superresolution" is meant precisely in the case of overcoming this threshold. Here it is important to have a correct a priori estimate of the uncertainty for each measuring experiment.

The complexity of the simultaneous influence of discretization and quantization on the evaluation of the experimental results is made obvious using this simple example. For more complex situations these effects will be concealed by other factors; the main sign of the influence under study, i.e., "unexpected" variations of the uncertainty of the result — will be preserved. Noise, especially a strong one, acts as a leveling factor.

The above discussion enables us to formulate the following view of the problem. An ideal realization quantization and discretization operation in measuring experiment is not the source of noise and consequently of errors. An "ideal" uncertainty of the measurement result appears; its properties differ from the properties of the error, and they may be used to improve the estimate of the experimental result. The "ideal" uncertainty becomes an error *only* (under ideal conditions) because of the use of the model (operation) of restoration of a measuring signal. The mechanism of this transition is triggered by assigning a certain value to the signal restored at the instant of sampling. Substantial information contained in the bounds of the interval number is lost. Any form of subsequent processing of information on the signal restored (filtration, identification) makes this loss irreversible. Under actual conditions, the above feature is concealed by the presence of noise. In the presence of a strong noise, this effect is difficult to use for improvement of the estimate of the measurement result. However, when the signal/noise ratio is high, wide opportunities for improvement of the measuring experiment are opened up [11, 12], which is proved by the numerical experiment in question.

We assume that, in addition to the standard types of uncertainties [13], it is expedient to add an uncertainty of type I that denotes the contribution of ideal sources, e.g., those of the type considered above, and is an integral part of an uncertainty of type A, along with the uncertainty introduced by random sources into it. The contribution of the "ideal" uncertainty may vary from the main contribution for a high signal/noise ratio to that barely noticeable for a high level of noise. Although, according to [13], it is formally proposed that an uncertainty of type I be classified as an uncertainty of type B, we assume that the "ideal" uncertainty deserves an individual status because of its origin and features of interaction with noise. In so doing, we somewhat resolve the contradiction between the error, the uncertainty, and an ideal measuring device.

The concept of error assumes the zero value of the error for the ideal measuring device under ideal conditions. For this reason, an uncertainty of type I cannot be classified as an error because of their different properties.

NOTATION

b and c, parameters of the model; f, functional dependence; m, number of samples; n, capacity of the analog-to-digital converter; p, arbitrary parameter; r, number of arrivals at the investigated range of uncertainty values; this number is normalized to the total number of experiments; t, running time of the experiment; U, reference voltage; x, quantity under study; y, measured quantity; Δ, uncertainty of the interval number; ν, quantum of the analog-to-digital converter; [], interval number. Subscripts: max, maximum; min, minimum; b, best; s, statistical.

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